What is a Hilbert Space?

Roughly speaking, a Hilbert Space is an infinite dimensional vector space with an inner product. Thus the topic can be considered as infinite dimensional linear algebra, but with one important exception: In linear algebra, one usually deals with finite sums, in Hilbert Spaces we want to be able to add up infinitely many vectors. This of course brings the question of convergence, and therefore a norm that will measure convergence. We also want series/ sequences that seem to be convergent to actually have a limit; a property called completeness. In short, Hilbert space techniques involve a combination of linear algebra and analysis. Some concepts (like metrics, convergence, completeness) are topics of Math 301; but for those who have not had 301, we will go over these concepts as much as necessary.

Content: Inner product, Hilbert space, examples, orthogonal expansions. Classical Fourier series; The Fejer kernel, Fejer's theorem, Parseval's formula, Weierstrass approximation theorem. Dual space, the Riesz-Frechet theorem. Linear operators, multiplication operators and infinite operator matrices, compact Hermitian and Hibert-Schmidt operators and the spectral theorem. Applications.

Text book : Introduction to Hilbert Space, Nicolas Young. (IC – reserve). The material will be shared in SuCourse+

Topics that we will covered (as much as possible) following the chapters of the book:

1. Inner product spaces

2. Normed spaces

3. Hilbert and Banach spaces

4. Orthogonal expansions

5. Classical Fourier series

6. Dual spaces

7. Linear operators

8. Compact operators

9. Sturm-Liouville systems

10. Green's functions

11. Eigenfunction expansions

12. Positive operators and contractions

There will be weekly homework assignments (40%), one midterm (30%) and a final, possibly take-home, (30%).